

Testing Gravity Against Early Time Integrated Sachs-Wolfe Effect

Pengjie Zhang^{1,2,*}

¹Shanghai Astronomical Observatory, Chinese Academy of Science, 80 Nandan Road, Shanghai, China, 200030

²Particle Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510-0500, USA

A generic prediction of general relativity is that the cosmological linear density growth factor D is scale independent. But in general, modified gravities do not preserve this signature. A scale dependent D can cause time variation in gravitational potential at high redshifts and provides a new cosmological test of gravity, through early time integrated Sachs-Wolfe (ISW) effect-large scale structure (LSS) cross correlation. We demonstrate the power of this test for a class of $f(R)$ gravity, with the form $f(R) = -\lambda_1 H_0^2 \exp(-R/\lambda_2 H_0^2)$. Such $f(R)$ gravity, even with degenerate expansion history to Λ CDM, can produce detectable ISW effect at $z \gtrsim 3$ and $l \gtrsim 20$. Null-detection of such effect would constrain λ_2 to be $\lambda_2 > 1000$ at $> 95\%$ confidence level. On the other hand, robust detection of ISW-LSS cross correlation at high z will severely challenge general relativity.

PACS numbers: 98.65.Dx, 95.30.Sf

Introduction.— Cosmological observations provide unique tools to study gravity at \gtrsim Mpc scales. General relativity, with the aid of the cosmological constant, or dark energy with equation of state $w \sim -1$, successfully reproduces the accelerated expansion of the Universe, indicated by SN Ia observations[1], along with the flatness of the Universe measured by the cosmic microwave background (CMB)[2] and distance measured by the baryon oscillations[3]. However, these observational evidences mainly constrain the mean expansion history of the Universe and can be reproduced by modified gravity such as brane world DGP theory [4] and generalized $f(R)$ gravity[5]. Essentially, the large scale structure (LSS) of the universe, such as weak gravitational lensing[6], is required to break this degeneracy.

General relativity imprints a unique signature in the LSS, which is scale *independent* linear density growth factor D at sub-horizon scale after matter-radiation equality epoch[7]. Modifications to general relativity not only changes the amplitude of D , but in general, causes D to be scale dependent. This unique feature of modified gravity has already been noticed in phenomenological theory of modified Newtonian potential[8] and in DGP[9]. It can be detected by weak gravitational lensing, galaxy clustering [8] and late time integrated Sachs-Wolfe (ISW) effect[9]. Counter-intuitively, in this paper, we show that modified gravity can produce a detectable *early time* ISW effect.

We investigate a class of $f(R)$ gravity with action

$$L = \int (R + f(R)) \sqrt{g} d^4x + L_{\text{matter}} , \quad (1)$$

and field equation

$$(1 + f_R) R_{uv} - \frac{g_{uv}}{2} (R + f - 2\Box f_R) - f_{R;u;v} = 8\pi G T_{uv} , \quad (2)$$

where $f_R \equiv df/dR$. We design $f(R) = -\lambda_1 H_0^2 \exp(-R/\lambda_2 H_0^2)$, where $\lambda_{1,2}$ are two positive dimensionless constants and H_0 is the Hubble constant at present. If we choose $\lambda_2 \ll R_{\text{solar}}/H_0^2 \sim \rho_{\text{solar}}/\rho_c \gtrsim 10^{10}$ (ρ_c is the critical density of the Universe), the exponential damping in $f(R)$ guarantees that there is no effect of $f(R)$ to our solar system and the exponential $f(R)$ can pass all solar system tests. For the $f(R)$ gravity, the application of Birkhoff theorem to perturbations of a spherically symmetric region leads to scale independent D [10]. But, as pointed out by [11], this approach may be problematic. We clarify this issue by solving the structure evolution of the fully covariant $f(R)$ gravity to linear order. We find that D shows nontrivial scale dependence[12].

The H - z relation of the $f(R)$ gravity.— Cosmological observations prohibit strong deviation of $f(R)$ from a cosmological constant. At the limit that $R(a \equiv 1/(1+z)) = 1) \ll \lambda_2 H_0^2$, the H - z relation of $f(R)$ gravity can have the same asymptotic behavior as that of Λ CDM. At low redshift where $R(a) \ll \lambda_2 H_0^2$, $f(R)$ behaves as a cosmological constant and the H - z relation resembles that of Λ CDM. At high redshifts where $R \gg \lambda_2 H_0^2$, $f(R) \rightarrow 0$ and $H(z) \rightarrow \Omega_0^{1/2} (1+z)^{3/2}$. Deviation from Λ CDM happens at some intermediate redshifts where $R(a) \sim \lambda_2 H_0^2$ and vanishes toward both higher and lower z . We quantify their difference by solving Eq. 2 of a flat universe to zero order

$$H^2 + \frac{f}{6} - \frac{\ddot{a}}{a} f_R + H \dot{f}_R = H_0^2 \Omega_0 a^{-3} . \quad (3)$$

This equation can be rewritten as $y = \Omega_0 - C(y(a))$, where $y \equiv a^3 H^2$, $C(y(a)) \equiv [f/6 - \ddot{a} f_R/a + H \dot{f}_R] a^3$ and Ω_0 is the dimensionless matter density at present.

*Electronic address: pjzhang@shao.ac.cn

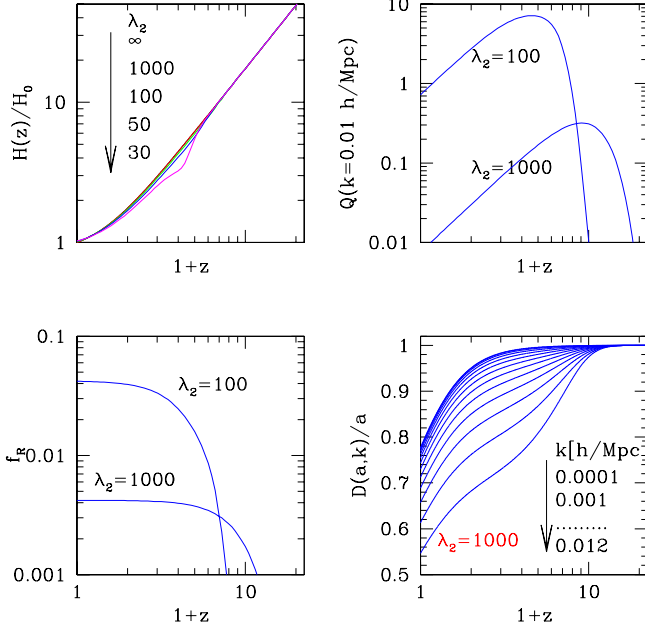


FIG. 1: The $H(z)$ - z relation and structure growth in the exponential $f(R)$ gravity. Top left panel: H - z . $\lambda_2 \rightarrow \infty$ corresponds to Λ CDM cosmology. Top right panel: $Q(k, a) \propto k^2$, which describes the main effect of $f(R)$ gravity to structure formation. We plot the result of $k = 0.01 h/\text{Mpc}$. Bottom left panel: $f_R(a)$, which determines the effective Newton's constant $G_{\text{eff}} = G/(1 + f_R)$. For $\lambda_2 \gtrsim 100$, its effect to structure formation can be neglected. Bottom right panel: $D(k, a)/a$ ($\lambda_2 = 1000$), where the linear density growth factor D is normalized such that $D \rightarrow a$ when $a \rightarrow 0$.

Since $C(y(a))$ is completely determined once y as a function of a is given, Eq. 3 can be solved iteratively by the iteration relation $y^{(i+1)} = \Omega_0 - C(y^{(i)})$. To mimic a Λ CDM universe, we fix λ_1 by requiring $f(R(a=1)) = -6H_0^2(1 - \Omega_0)$. The iteration converges quickly by taking the initial guess $y^{(0)} = \Omega_0 + (1 - \Omega_0)a^3$. For $\lambda_2 \geq 100$, $y^{(1)}$ is accurate to $\sim 1\%$. As expected, for $\lambda_2 \geq 100$, the $H(z)$ - z relation is almost identical to the corresponding Λ CDM cosmology (Fig. 1). Such $f(R)$ gravity can not be distinguished from Λ CDM by inflation, big bang nucleosynthesis (BBN), primary CMB, SN Ias and other measures of H - z relation.

The large scale structure of the $f(R)$ gravity.— We will show that, even with this degeneracy in H - z relation and solar system behavior, the LSS of the $f(R)$ gravity could be significantly different to that of Λ CDM. We choose the Newtonian gauge

$$ds^2 = -(1 + 2\psi)dt^2 + a^2(1 + 2\phi)dx^{i,2}. \quad (4)$$

There are four perturbation variables ϕ , ψ , the matter over-density δ and the (comoving) peculiar velocity convergence θ .

In general relativity, $\phi = -\psi$, as long as there is no

anisotropic stress. But in modified gravity, this relation breaks in general. ij ($i \neq j$) component of Eq. 2 provides the relation between ϕ and ψ . For $f(R)$ gravity, due to non-vanishing $f_{R;ij}$ ($i \neq j$), ϕ - ψ relation becomes scale dependent. Throughout this paper, we neglect time derivative terms with respect to spatial derivative terms of corresponding variables. This simplification holds at scales $k \gtrsim aH \lesssim 10^{-3}h/\text{Mpc}$. Since we will focus on the ISW effect at $l \gtrsim 20$ and $z \gtrsim 3$ where the relevant $k \gtrsim 5 \times 10^{-3}h/\text{Mpc}$, this simplification is sufficiently accurate. We then obtain

$$\phi + \psi = \frac{f_{RR}c^2}{1 + f_R} \frac{2}{a^2} (\nabla^2 \psi + 2\nabla^2 \phi). \quad (5)$$

In Fourier space, this reads $\psi = -\phi(1 - 2Q)/(1 - Q)$, where $Q(k, a) \equiv -2f_{RR}c^2k^2/(1 + f_R)a^2$ and $f_{RR} \equiv d^2f/dR^2$. For clarity, we explicitly show the speed of light c . We will see that this scale dependent ψ - ϕ relation has profound effect on the LSS. Combining Eq. 5 and the tt component of Eq. 2, we obtain the new Poisson equation

$$\nabla^2(\phi - \psi) = -\frac{3H_0^2\Omega_0}{1 + f_R}a^{-1}\delta. \quad (6)$$

The energy-momentum tensor is still conserved and provides the remaining two equations:

$$\dot{\delta} + \theta = 0, \quad \dot{\theta} + 2H\theta + \frac{1}{a^2}\nabla^2\psi = 0. \quad (7)$$

Combining all 4 equations, we obtain the main equation of this paper:

$$\delta'' + \delta' \left(\frac{3}{a} + \frac{H'}{H} \right) - \frac{\delta}{a^2} \frac{1 - 2Q}{2 - 3Q} \frac{3H_0^2\Omega_0}{a^3H^2(1 + f_R)} = 0, \quad (8)$$

where $' \equiv d/da$. In general relativity, $Q = 0$, so D is scale independent at scales $k \gtrsim aH/c$, no matter what the form of dark energy is. But in $f(R)$, the scale dependent $Q(k, a)$ induces nontrivial scale dependence to D . This behavior can not be obtained by a simple change in the effective Newton's constant. Furthermore, the correction Q has a nontrivial dependence on a . This is hard to realize by simply changing the form of the Newtonian potential (e.g. to Yukawa potential).

Since $f_{RR} < 0$, there exist one *apparent singularity* $Q = 2/3$ in Eq. 8, where only $\delta = 0$ solution is accepted and two at $Q = 1/2, 1$ in the ψ - ϕ relation, where only $\psi = \phi = 0$ solution is accepted. We leave this issue alone until the discussion section. For the moment, we take a modest goal by only using regions where $Q < 1/2$ to constrain $f(R)$. For $\lambda_2 = 1000$, this constrains us to region where $k \leq 0.012h/\text{Mpc}$.

Hereafter, we fix $\lambda_2 = 1000$. At $z \gg 1$, $H \propto a^{-3/2}$, $D \propto a^{1-\eta}$ when $\eta \equiv 3Q/5(2 - 3Q) \ll 1$. Thus gravitational potential decays at high redshifts with rate $\propto a^{-\eta}$

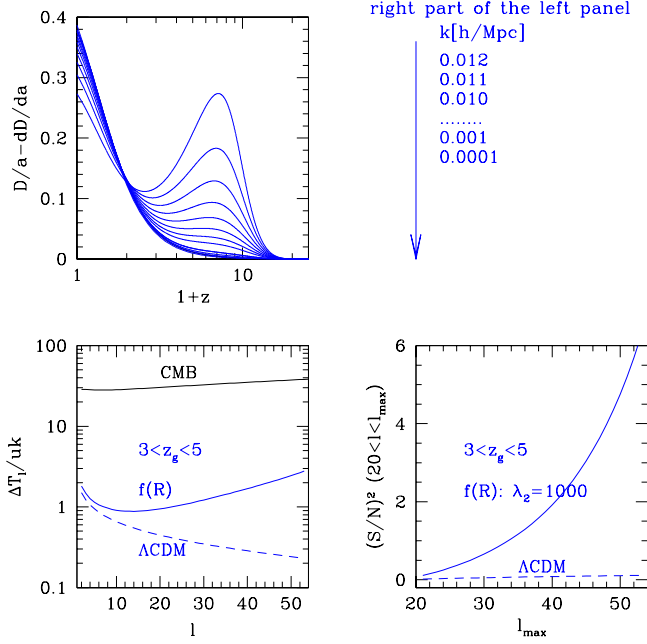


FIG. 2: The ISW effect. $\lambda_2 = 1000$ is adopted. Top left panel: $D/a - dD/da$, which determines the sign and amplitude of the ISW effect. D is normalized such that $D \rightarrow a$ when $a \rightarrow 0$. Bottom left panel: the ISW effect. Bottom right panel: Cumulative S/N of the ISW-LSS cross correlation measurements.

and causes an observable integrated Sachs-Wolfe (ISW) effect. At later time when $R \lesssim \lambda_2 H_0^2$, $Q \rightarrow 0$ (Fig. 1), the evolution of D approaches that of ΛCDM . For the exponential $f(R)$, $Q(a)$ peaks at $z \gg 1$ (Fig. 1), so the resulting ISW effect peaks at $z \gg 1$, as contrast to that of ΛCDM cosmology or dark energy models with $w \sim -1$. This provides us a unique way to test this form of $f(R)$. We solve Eq. 8 numerically. Initial condition is set to normalize $D \rightarrow a$ when $a \rightarrow 0$.

The integrated Sachs-Wolfe effect. — Time variation in $\psi - \phi$ causes a fractional CMB temperature variation [14]

$$\frac{\Delta T}{T_{\text{CMB}}} = \int [\dot{\psi} - \dot{\phi}] a d\chi. \quad (9)$$

Here, χ is the comoving angular diameter distance. Since both $\psi - \phi$ and the LSS trace the underlying matter distribution, there exists an ISW-LSS cross correlation, with power spectrum

$$\frac{l^2}{2\pi} C_l^{\text{ISW-LSS}} = \frac{\pi}{l} \int \Delta_{(\psi-\phi)\delta_{\text{LSS}}}^2 \left(\frac{l}{\chi}\right) W_{\text{LSS}}(\chi) a^2 \chi d\chi. \quad (10)$$

Here, δ_{LSS} is the density fluctuation of the LSS tracers, W_{LSS} is the corresponding weighting function and $\Delta_{(\psi-\phi)\delta_{\text{LSS}}}^2$ is the corresponding 3D power spectrum(variance). The above formula adopts the Limber's

approximation, which is sufficiently accurate to serve for our interest at $l \geq 20$. The amplitude and sign of the ISW effect is determined by $A_{\text{ISW}} \equiv D/a - dD/da$. Positive A_{ISW} means positive correlation between ISW and LSS. For $k \gtrsim 0.007 h/\text{Mpc}$, A_{ISW} has a bump at $z \sim 6$, whose amplitude increases towards small scales (large k). This boosts early time small scale ISW signal (Fig. 2).

Here we estimate the S/N of the ISW-LSS cross correlation measurements. Since the exponential $f(R)$ does not affect physics at $z \gtrsim 100$, we adopt the same primordial power spectrum with power index $n = 1$, the same transfer function BBKS [15] and the same amplitude at $a_i = 0.01$, as that of the ΛCDM cosmology. The LSS tracers we choose are 21cm emitting galaxies at $3 < z < 5$, which will be measured by proposed 21cm experiments such as Square Kilometer Array[16]. Singularities presented in the perturbation equations limit us to $l < 60$, where one can neglect shot noises of both the CMB and LSS. At this limit, the S/N of each l is

$$\left(\frac{S}{N}\right)^2 \simeq \frac{(2l+1)f_{\text{sky}}r^2}{1 + C_l^{\text{CMB}}/C_l^{\text{ISW}}} \quad (11)$$

where C_l^{CMB} , C_l^{ISW} and r are the primary CMB power spectrum, ISW power spectrum and the cross correlation coefficient between ISW and LSS, respectively. Since r has very weak dependence on galaxy bias, the estimation presented here is weakly model dependent. We disregard signals from $l < 20$, to reduce confusions of ΛCDM cosmology or dark energy models. For sparse galaxy sampling which is sufficient for our purpose, SKA is able to cover the whole sky. So we assume that $f_{\text{sky}} = 1$. The cumulative $\sum_{20}^{l_{\text{max}}} (S/N)^2$ is shown in Fig.2.

The ISW signal peaks at $z \gtrsim 3$ and increases toward high l . This is hard to mimic by ΛCDM , dark energy or many forms of modified gravity. (1) For ΛCDM or dark energy models with $w \lesssim -1$, at $z \gtrsim 3$, the ISW effect effectively vanishes. Fig. 2 shows that ΛCDM can be distinguished from the $\lambda_2 = 1000$ $f(R)$ gravity with $> 2\sigma$ confidence by the ISW-21cm emitting galaxy cross correlation. (2) For dark energy models with $w \gtrsim -1$, A_{ISW} does not decrease as fast as that of ΛCDM . But the ISW signal (including contributions from dark energy fluctuations) decreases toward high l [17] and one does not expect a detectable ISW effect. (3) For DGP, a negative ISW-LSS cross correlation may be expected[9]. (4) For generalized $f \propto (\alpha R^2 + \beta R_{ab}R^{ab} + \gamma R_{abcd}R^{abcd})^{-n}$ ($n > 0$), the ISW effect vanishes at high z because the f correction decreases much faster than the exponential $f(R)$. So we expect that null detection of ISW-LSS cross correlation at $l \geq 20$ and $z \geq 3$ would constrain λ_2 to $\lambda_2 > 1000$ at $> 2\sigma$ confidence level. On the other hand, a detection of such cross correlation would present as a severe challenge to general relativity.

Discussion. — The scale dependence of D , as an unambiguous signature of modified gravity, can in principle be

measured from weak gravitational lensing by the mean of lensing tomography. Since ϕ is no longer equal to $-\psi$, we provide the general form of the lensing transformation matrix A_{ij}

$$A_{ij} - \delta_{ij} = \int_0^{\chi_s} d\chi (\phi - \psi)_{,ij} W(\chi, \chi_s), \quad (12)$$

where $W(\chi, \chi_s) = \chi(1 - \chi/\chi_s)$ is the usual lensing kernel. All basic lensing theorems remain unchanged. For example, lensing shear field is still curl free (*if neglecting second order corrections such as Born correction*). For $f(R)$ gravity, relation between the lensing convergence $\kappa = 1 - (A_{11} + A_{22})/2$ and the matter over-density resembles that of the general relativity, with

$$\kappa = \frac{3}{2} H_0^2 \Omega_0 \int \delta a^{-1} W(\chi, \chi_s) (1 + f_R)^{-1} d\chi. \quad (13)$$

It is interesting to see how well weak lensing alone can constrain modified gravity. For the exponential $f(R)$, one complexity is that lensing mainly probes LSS at $z \lesssim 1$, where Q is small and the deviation from a scale independent D is small, so the constraints may be weak. This can be significantly improved by gravitational potential reconstructed from primary CMB. Combining lensing and CMB measurements, it is very promising to measure the evolution of the gravitational potential between $z = 1100$ and $z \sim 0$ robustly. This will put strong constraints on the nature of gravity. Unfortunately, due to singularities in the perturbation equations, we are limited to scales $k \lesssim 0.012 h/\text{Mpc}$ or $l \lesssim 20$ at $z \lesssim 1$ (for $\lambda_2 = 1000$). Information contained in this region is very limited and could be contaminated by other physics such as dark energy fluctuations. Solving the field equation crossing those singularities consistently is nontrivial. We leave this work for future study.

The $Q = 1/2, 2/3, 1$ singularities may be caused by awkward gauge choice, the neglecting of time derivative terms with respect to corresponding spatial derivative terms, or the failure of the perturbation approach. For example, for $Q = 2/3$, the only solution $\delta = 0$ does not depend on initial conditions. This could be caused by neglecting time derivative times, which erases some degree of freedom. These issues require detailed study. But if these singularities do exist, the LSS can rule out most $f(R)$ gravities as alternatives to dark energy or general relativity, because the existence of singularities in the perturbation equations is generic in many, if not all, $f(R)$ gravities which can reproduce the expansion history of the Universe. To produce a similar expansion history, (1) R should increase when a decreases and (2) $f(R(a=1)) < 0$ in order to have an accelerated expansion at present and $f(R(a \rightarrow 0)) \rightarrow 0$ in order not to affect inflation, BBN and primary CMB. This results in $f_R(a) > 0$ at least at some early epoch a_+ . To avoid singularity, $f_{RR} \geq 0$ must be satisfied at all epochs. Thus

$f_R(a < a_+) \geq f_R(a_+) > 0$. On the other hand, when $a \rightarrow 0$, $R \propto a^{-3}$ and increases very quickly. So, f increases toward high redshift and crosses over zero at some epoch and then increases faster than a^{-3} . This contradicts our expectation.

To demonstrate the power of LSS to constrain gravity, we adopt a conservative requirement to avoid singularities at $k < k_s$. At the limit that $\lambda_2 \gg 1$, Q peaks at $a = (2\lambda_2/9\Omega_0)^{-1/3}$ and the peak amplitude is $\simeq 12(1 - \Omega_0)(2/9\Omega_0 e)^{2/3} \lambda_2^{-4/3} (ck/H_0)^2$, where we show the speed of light c explicitly. To avoid singularities at $k < k_s$,

$$\lambda_2 \geq 2.5 \times 10^5 \left(\frac{k_s}{h/\text{Mpc}} \right)^{3/2} \quad (14)$$

should be satisfied.

Acknowledgment.— I thank Jochen Weller for providing the Runge-Kutta code dverk.F and John Barrow, Xuelei Chen and Tomi Koivisto for helpful discussions. This work was supported in part by the One-Hundred-Talent Program of China and by the DOE and the NASA grant NAG 5-10842 at Fermilab.

-
- [1] A.G. Riess et al., 2004, ApJ, 665; And references therein
 - [2] D.N. Spergel, et al. 2003, ApJS, 148, 175
 - [3] D.J. Eisenstein, et al. 2005, astro-ph/0501171
 - [4] Dvali, G., Gabadadze, G. & Porrati, M. 2000, Physics Letters B, 485, 208
 - [5] S.M. Carroll, V. Duvvuri, M. Trodden & M.S. Turner 2004, PRD, 70, 043528; S.M. Carroll, A. de Felice, V. Duvvuri, D.A. Easson, M. Trodden, & M.S. Turner, 2005, PRD, 71, 063513; O. Mena, J. Santiago & J. Weller, 2005, astro-ph/0510453
 - [6] L. Knox, Y.S. Song & J.A. Tyson, 2005, astro-ph/0503644; M. Ishak, A. Upadhye & D.N. Spergel, 2005, astro-ph/0507184
 - [7] Neutrino free streaming causes D to be scale dependent, but this effect is small. Fluctuations in dark energy cause D to be scale dependent at \sim horizon scale. But this effect vanishes toward smaller scales.
 - [8] M. White, C.S. Kochanek, 2001, ApJ, 560, 539; C. Seal-fon, L. Verde & R. Jimenez, 2005, PRD, 71, 083004
 - [9] I. Sawicki, S.M. Carroll, 2005, astro-ph/0510364
 - [10] M. Amarzguoui, O. Elgaroy, D.F. Mota, T. Multamaki. 2005, astro-ph/0510519
 - [11] Y. Song, 2005, PRD, 71, 024026; T. Koivisto, H. Kurki-Suonio, F. Ravndal, 2005, PRD, 71, 064027
 - [12] [13] also found scale dependent D using the Palatini formulation. But their results are hard to compare with ours quantitatively.
 - [13] T. Koivisto & H. Kurki-Suonio, 2005, astro-ph/0509422
 - [14] R.K. Sachs & A.M. Wolfe, 1967, ApJ, 147, 73
 - [15] J.M. Bardeen, J.R. Bond, N. Kaier & A.S. Szalay, 1986, ApJ, 304, 15
 - [16] <http://www.skatelescope.org/>
 - [17] R. Bean & O. Dore, 2004, PRD, 69, 083503; W. Hu & R. Scranton, 2004, PRD, 70, 123002